

The Second International Verification of Neural Networks Competition (VNN-COMP 2021): Summary and Results

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Abstract

This report summarizes the second International Verification of Neural Networks Competition (VNN-COMP 2021), held as a part of the 4th Workshop on Formal Methods for ML-Enabled Autonomous Systems that was collocated with the 33rd International Conference on Computer-Aided Verification (CAV). Twelve teams participated in this competition. The goal of the competition is to provide an objective comparison of the state-of-the-art methods in neural network verification, in terms of scalability and speed. Along this line, we used standard formats (ONNX for neural networks and VNNLIB for specifications), standard hardware (all tools are run by the organizers on AWS), and tool parameters provided by the tool authors. This report summarizes the rules, benchmarks, participating tools, results, and lessons learned from this competition.

1 Introduction

Methods based on machine learning are increasingly being deployed for a wide range of problems, including recommendation systems, machine vision and autonomous driving. While machine learning has made significant contributions to such applications, few tools provide formal guarantees about the behaviours of neural networks.

In particular, for data-driven methods to be usable in safety-critical applications, including autonomous systems, robotics, cybersecurity, and cyber-physical systems, it is essential that the behaviours generated by neural networks are well-understood and can be predicted at design time. In the case of systems that are learning at run-time it is desirable that any change to the underlying system respects a given safety-envelope for the system.

While the literature on verification of traditionally designed systems is wide and successful, there has been a lack of results and efforts in this area until recently. The International Verification of Neural Networks Competition (VNN-COMP) was established in 2020, aiming to bring together researchers working on techniques for the verification of neural networks. In 2021, VNN-COMP¹ was held as a part of the 4th Workshop on Formal Methods for ML-Enabled Autonomous Systems (FoMLAS) that was collocated with the 33rd International Conference on Computer-Aided Verification (CAV).

While the first VNN-COMP in 2020 was a friendly competition where the participants tested their tools and reported the results in parallel, the second VNN-COMP in 2021 aims to provide a fair comparison and standardize the pipeline of the competition. Such standardization includes 1) standard formats where we use ONNX for neural networks and VNNLIB for specifications; and 2) standard hardware where all tools are run by the organizers on AWS either on CPU instances or cost-equivalent GPU instances. The competition was kicked off in January and the solicitation for participation was sent in February 2021. By March, several teams registered

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¹<https://sites.google.com/view/vnn2021/home>

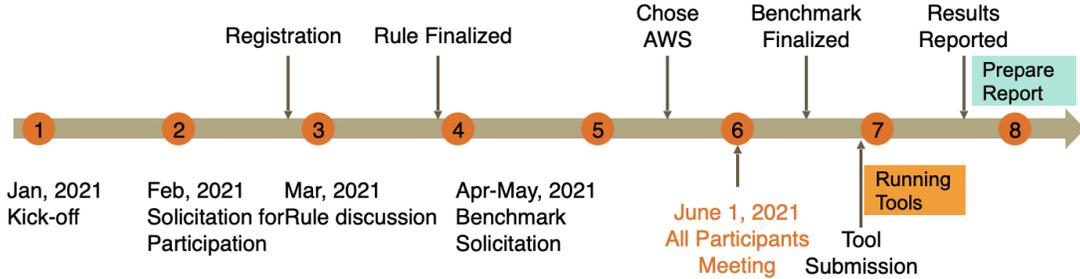


Figure 1: Timeline for VNN-COMP2021.

to participate in this competition. The rule discussion was finalized in March 2021 where the finalized rules are summarized in section 2. From April to May 2021, the benchmarks to test the tools were solicited. Meanwhile, after comparing different choices, the organizing team finally decided to use AWS as our testing platform. On June 1st, 2021, all participants had an online meeting to agree on the rules and benchmarks. By the end of June 2021, twelve teams submitted their tools and the organizers spent two weeks running the tools on AWS to obtain the final results. The final results were reported in FoMLAS on July 19, 2021. The timeline is summarized in fig. 1. Most discussions took place on the repository <https://github.com/stanleybak/vnncomp2021>. There are three issues: rules discussion, benchmarks discussion, and tool submission, serving as the venues for the corresponding discussions. Moreover, the repository hosts all the submitted benchmarks and the scripts to run the tests.

The remainder of this report is organized as follows. Section 2 discusses the competition rules. Section 3 lists all participating tools, Section 4 lists all benchmarks, and Section 5 summarizes the results. Section 6 concludes the report and discusses potential future improvements.

2 Rules

Terminology An *instance* is defined as (specification (pre- and post-condition), network, timeout). For example: an MNIST classifier with one input image, a given local robustness threshold ϵ , and a specific timeout. A *benchmark* is defined as a set of instances. For example: a specific MNIST classifier with 100 input images, a given robustness threshold ϵ , and one timeout per input.

Run-time caps Per instance: any verification instance will timeout after at most X minutes, determined by the benchmark proposer. These can be different for each instance. Per benchmark: there is an upper runtime limit of 6 hours per benchmark. For example, a benchmark proposal could have six instances with a one hour timeout, or 100 instances each with a 3.6 minute timeout. To provide a fair comparison, we quantify the startup overhead for each tool by running it on small networks; and then we subtract the overhead from the total runtime.

Instance score Each instance is scored as follows:

- Correct hold: 10 points;
- Correct violated (where random tests or simple adversarial example generation did not succeed): 10 points;

- Correct violated (where random tests or simple adversarial example generation succeeded): 1 point;
- Incorrect result: -100 points.

Time bonus Time bonus is computed as follows.

- The fastest tool for each solved instance will receive +2 points.
- The second fastest tool will receive +1 point.

All runtimes below 1.0 seconds after overhead correction (explained below) are considered to be 1.0 seconds exactly for scoring purposes. If two tools have runtimes within 0.2 seconds (after all corrections), for scoring purposes we will consider them the same runtime.

Overhead Correction According to the rules discussion, we decided to subtract tool overhead time from the results. For example, simply importing tensorflow from Python and acquiring the GPU can sometimes take about 5 seconds, which would be unfortunate for benchmarks like ACASXu where some verification times are under a second.

To subtract overhead, we created trivial network instances and included those in the measurements. We then observed the minimum verification time along all instances, and considered that to be the overhead time for the tool.

One issue with that was that some tools had different overhead depending on if they were run in CPU mode or GPU mode, and this type of measurement penalized the GPU mode unintentionally. In the score reporting, we include a multi-overhead result where we apply the overhead measured for the mode the tool was actually run in.

Benchmark score The benchmark score of each category is a percentage. It is computed as 100 times the sum of the individual instance scores for that benchmark category divided by the maximum sum of instance scores of any tool for that benchmark category. For example, the tool with the highest sum of instance scores for a category should get 100%.

Format This year we standardized the inputs to be `onnx` neural networks and `vnnlib` specification files. Tool authors were also required to provide scripts to install their tool as much as possible, as well as run their tool on a specific instance provided the network file, specification file, and timeout. Specifications included simple disjunctions in both pre- and post-conditions to encode properties like an unchanged classification for an input in one of multiple hyper-boxes. The specification is regarded as encoding a counter-example, meaning that a property is proven “correct” if the specification is shown to be unsatisfiable, while the property is shown to be violated, if a counterexample fulfilling the specification is found. Hence, robustness with respect to inputs in a hyper-box has to be encoded as disjunctive property, where any of the other classes is constrained to become the maximum output.

3 Participants

The following tools and teams participated in VNN-COMP. They are summarized in table 1.

3.1 Marabou

Team Guy Amir¹, Clark Barrett², Ahmed Irfan³, Guy Katz¹, Teruhiro Tagomori⁴, Alex Usvyatsov¹, Haoze Wu², Aleksandar Zeljic².

¹ Hebrew University of Jerusalem, ² Stanford University, ³ Amazon Web Services, ⁴ NRI Secure.

Tool	GPU?	Floating Point Accuracy	Use of External Solvers
Marabou	-	LP-default	Gurobi
VeriNet	-	Either	Xpress Solver
ERAN	Yes	Sound	Gurobi
α, β -CROWN	Yes	Either	Gurobi
DNNF	-	Either	None
NNV	-	64bit	Matlab
OVAL	Yes	Either	None
RPM	-	64bit	None
NV.jl	-	64bit	None
Venus	-	64bit	Gurobi
Debona	-	32bit	Gurobi
mnenum	-	Either	None

Table 1: Summary of key features of participating tools. For the “GPU” column, we only mark the tools that were tested on GPU during this competition. Tools that support GPU but were tested on CPU during this competition are not marked. For the floating point accuracy, “either” means the tool can handle either 32 or 64 bit floating point, depending on the onnx network; “LP-default” means the tool uses the default settings for LP solver; “sound” means the tool is fully floating-point-sound arithmetic with respect to IEEE-754 semantics up to the LP solver (up to 64x slower than non-fp-sound 32 bit arithmetic).

Description Marabou [22] is a Neural Network Verification toolkit that can answer queries about a network’s properties by encoding and solving these queries as constraint satisfaction problems. It can accommodate networks with different activation functions (including ReLU, Leaky ReLU, Sign [1], Max, Absolute value) and topologies (e.g., FFNNs, CNNs, residual connections). It also uses the Split-and-Conquer algorithm [51] for parallelization to further enhance scalability. Marabou accepts multiple input formats, including protocol buffer files generated by the popular TensorFlow framework for neural networks, and the ONNX format.

The core of Marabou resolves around the Reluplex procedure [21], but it also supports multiple new techniques and solving modes. In particular, it incorporated the DeepPoly analysis introduced in [38] and the (MI)LP-based bound tightening first seen in [42]. In terms of complete verification procedure, in addition to the Reluplex procedure, Marabou also supports solving the verification query with a MILP encoding.

For the competition, Marabou uses the DeepPoly analysis to tighten the variable bounds and then uses a portfolio strategy for complete verification, with a fraction of the CPUs solving a MILP-encoding of the verification problem with Gurobi, and the rest running a new complete verification procedure that is currently under submission.

Link <https://github.com/NeuralNetworkVerification/Marabou>

Commit For reproduction of VNN-Comp results, use https://github.com/anwu1219/Marabou_private/commit/81e9f14f7ae9f6a2097524ea1291e86434ef42dc.

Hardware and licenses CPU, Gurobi License.

Participated benchmarks ACASXu, cifar10_resnet, eran, marabou-cifar10, mnistfc, oval21, verivital.

3.2 VeriNet

Team Patrick Henriksen, Alessio Lomuscio (Imperial College London).

Description VeriNet [17, 18] is a complete Symbolic Interval Propagation (SIP) based verification toolkit for feed-forward neural networks. The underlying algorithm utilises SIP to create a linear abstraction of the network, which, in turn, is used in an LP-encoding to analyse the verification problem. A branch and bound-based refinement phase is used to achieve completeness.

VeriNet implements various optimisations, including a gradient-based local search for counterexamples, optimal relaxations for Sigmoids, adaptive node splitting [17], succinct LP-encodings, and a novel splitting heuristic that takes into account indirect effects splits have on succeeding relaxations [18].

VeriNet supports a wide range of layers and activation functions, including Relu, Sigmoid, Tanh, fully connected, convolutional, max and average pooling, batch normalisation, reshape, crop and transpose operations, as well as additive residual connections.

Note that VeriNet subsumes the Deepsplit method presented in [18].

Link Scheduled for release late August 2021 at: <https://vas.doc.ic.ac.uk/software/neural/>.

Hardware and licences CPU and GPU, Xpress Solver license required for large networks.

Participated benchmarks ACASXu, cifar10_resnet, cifar2020, eran, marabou-cifar10, mnistfc, nn4sys, oval21, verivital.

3.3 ERAN

Team Mark Niklas Müller (ETH Zurich), Gagandeep Singh (UIUC), Markus Püschel (ETH Zurich), Martin Vechev (ETH Zurich)

Description ERAN [30, 33, 36–39] is a neural network verifier based on leveraging abstract interpretations to encode a network, pre- and post-condition as an LP or MILP problem. ERAN supports both incomplete and complete verification and can handle fully-connected, convolutional, and residual network architectures containing ReLU, Sigmoid, Tanh, and Maxpool non-linearities. It uses 64-bit precision (up to 16 times slower than 32-bit on some GPUs) and an arithmetic which is floating-point-soft (performs 4 times more operations) with respect to IEEE-754 semantics up to the LP solver. Single- and multi-neuron relaxations of non-linear activations [30] computed using GPUPoly [33] are combined with a partial MILP encoding and neuron-wise bound-refinement [39] to obtain a precise network encoding. The analyzer is written in Python, uses ELINA [40] for numerical abstractions, and Gurobi for solving LP and MILP instances. While the use of both GPU and CPU enables ERAN to utilize all resources of a system it also requires both a relatively strong GPU and CPU to be available in order to avoid one bottlenecking the other (which occurred with the GPU AWS instance).

When run in complete mode, ERAN generates concrete counterexamples. In incomplete mode, ERAN attempts to falsify a property by running a PGD attack before attempting verification using increasingly more expensive and precise abstractions. As we use the same abstractions for all instances of a benchmark, ERAN might fail to verify a property before exceeding the timeout. In these cases, a more expensive abstraction might have been able to verify the property in time.

Link <https://github.com/eth-sri/eran>

Commit Please use the main repository for anything but reproducing VNN-COMP results.

https://github.com/mnmueller/eran_vnncomp2021.git

(1e474c77f72f86f450df9f0a860b4d35c490ea7c)

Hardware and licences CPU and GPU, GUROBI License

Participated benchmarks ACASXu, cifar10_resnet, cifar2020, eran, marabou-cifar10, mnistfc, nn4sys, oval21, verivital.

3.4 α, β -CROWN

Team Huan Zhang* (Carnegie Mellon), Kaidi Xu* (Northeastern), Shiqi Wang* (Columbia), Zhouxing Shi (UCLA), Yihan Wang (UCLA), Xue Lin (Northeastern), Suman Jana (Columbia), Cho-Jui Hsieh (UCLA), Zico Kolter (Carnegie Mellon); * indicates equal contribution.

Description The α, β -CROWN (**alpha-beta-CROWN**) verifier is based on an efficient bound propagation algorithm, CROWN [55], with a few crucial extensions [49, 53, 54]. We use the generalized version of CROWN in the `auto_LiRPA` library [53] which supports general neural network architectures (including convolutional layers, residual connections, recurrent neural networks and Transformers) and a wide range of activation functions (e.g., ReLU, tanh, sigmoid, max pooling and average pooling), and is efficiently implemented on GPUs. We jointly optimize intermediate layer bounds and final layer bounds using gradient ascent (referred to as α -CROWN or optimized CROWN/LiRPA [54]). Additionally, we use branch and bound [10] (BaB) and incorporate split constraints in BaB into the bound propagation procedure efficiently via the β -CROWN algorithm [49]. The combination of efficient, optimizable and GPU accelerated bound propagation with BaB produces a powerful and scalable neural network verifier.

Our verifier also utilizes a mixed integer programming (MIP) solver (Gurobi) for networks where MIP runs relatively fast, following the formulation in [42]. We use MIP to solve the tightest possible intermediate layer bounds for as many neurons as possible on CPUs within the timeout budget, and use α -CROWN to solve the remaining ones on GPUs. Finally, we conduct BaB with β -CROWN using tightened bounds. Although the GPU AWS instance has weak CPUs, we still find that MIP is helpful for some benchmarks, and it can become more beneficial on a machine with both strong CPUs and GPUs. Note that α -CROWN can exceed the power of a typical LP verifier when intermediate layer bounds are jointly optimized [32, 54], so we do not use a LP solver to tighten bounds.

Link <https://github.com/huanzhang12/alpha-beta-CROWN>

Commit c12e6eeaf6b16f99a99b65f377d0f450d6466a83 (only for reproducing competition results; please use the `main` branch version for other proposes)

Hardware and licenses CPU and GPU with 32-bit or 64-bit floating point; Gurobi license required for `mnistfc`, `eran`, `marabou-cifar10` and `verivital` benchmarks.

Participated benchmarks ACASXu, cifar10_resnet, cifar2020, eran, marabou-cifar10, mnistfc, nn4sys, oval21, verivital.

3.5 DNNF

Team David Shriver, Sebastian Elbaum, Matt Dwyer (University of Virginia).

Description DNNF [35] is a tool for neural network property falsification. It only attempts to find counter-examples to a property specification, and will not prove that a property holds. DNNF reduces properties and networks to robustness problems which can then be falsified using many different adversarial attack methods. Our reduction approach enables us to support much more complex property and network specifications, such as specifications with direct input and output relations, such as the relation $y > x$. The backend falsification method used for VNN-COMP is a custom implementation of PGD, but DNNF can also be run with several off-the-shelf

methods from foolbox or cleverhans, two popular python packages for adversarial attacks.

DNNF makes use of the DNNV framework [34] to load networks and properties, as well as to perform network simplifications. The current implementation of DNNF supports many different network operations. In particular it supports ONNX models with the following operations: Add, Atan, AveragePool, BatchNormalization, Concat, Conv, ConvTranspose, Elu, Flatten, Gather, Gemm, LeakyRelu, MatMul, MaxPool, Mul, Relu, Reshape, Resize, Shape, Sigmoid, Softmax, Sub, Tanh, Transpose, Unsqueeze.

DNNF can also be run on GPUs, which can speed up falsification for large models.

Link <https://github.com/dlshriver/DNNF>

Commit VNN-COMP results can be reproduced with commit `d4f08b43e4ad622157c65ac071183a3a0f4e6fe0`. For other uses, we suggest the `main` branch.

Hardware and licences DNNF can be run on the CPU or GPU, with no additional licenses required.

Participated benchmarks ACASXu, cifar10_resnet, cifar2020, eran, marabou-cifar10, mnistfc, nn4sys, oval21, verivital.

3.6 NNV

Team Neelanjana Pal (Vanderbilt University), Taylor T Johnson (Vanderbilt University)

Description The Neural Network Verification Tool (NNV) [43–46, 52] is written primarily with Matlab and implements reachability-analysis methods for neural network verification with a particular focus on applications of closed-loop neural network control systems in autonomous cyber-physical systems. NNV uses geometric representations such as star sets that allows for a layer-by-layer computation of the exact reachable set for feed-forward deep neural networks. In the event that a particular safety property is violated, NNV can construct and visualize the complete set of counterexample inputs for a neural network.

Link <https://github.com/verivital/nnv>.

Commit `3ca2629aaceb9080e4d08a0f9c6b51854f9c7b7f` (for reproducing competition results; otherwise please use the master version).

Hardware and licences GPU and CPU with 64-bit floating point. A license for Matlab will be required.

Participated benchmarks ACASXu, cifar2020, eran, mnistfc, oval21, verivital.

3.7 OVAL

Team Alessandro De Palma (University of Oxford), Florian Jaeckle (University of Oxford), M. Pawan Kumar (University of Oxford)

Description The OVAL verification tool is an optimization-based complete verifier that relies on a specialized Branch and Bound (BaB) framework for neural network verification. In this context, a BaB method is composed of three main components (see [9] for an overview): a *branching strategy* to divide the verification property into easier subproblems, a *bounding algorithm* to compute over-approximation bounds for each subproblem, and a *falsification algorithm* to look for counter-examples to the property.

For networks with medium to large input dimensionality, OVAL relies on the efficient FSB [13] branching strategy, which combines a dual-based scoring of ReLU neurons [9] with inexpensive strong branching approximations to select an activation to split upon. For networks of small input dimensionality, OVAL can revert to input splitting as in [10]. Bounds on the

subproblems are obtained by adaptively choosing [12] between bounding algorithms of varying degrees of tightness: the competition entry relies on Beta-CROWN [49], which effectively solves the convex hull of element-wise activations, and Active Set [11], which operates on the tighter relaxation from [2] to tackle harder properties. In addition, the framework supports a variety of bounding algorithms [8, 12, 15, 54]. The search for counter-examples is performed using the MI-FGSM [14] adversarial attack, which we adapted to perform general property falsification.

The implementation of the OVAL framework, written in PyTorch [31], exploits GPU acceleration and is massively parallel over both the BaB subproblems and the relative intermediate computations [8]. It currently supports fully connected and convolutional networks, with ReLU, maxpool and average pooling layers.

Link <https://github.com/oval-group/oval-bab>.

Commit `014b6ee5071508430c8e515bbae725306db68fe1` in order to reproduce competition results. We otherwise suggest to employ the master version.

Hardware and licences GPU and CPU with 32-bit or 64-bit floating point. No license is required.

Participated benchmarks ACASXu, cifar2020, eran, marabou-cifar10, mnistfc, nn4sys, oval21, verivital.

3.8 RPM

Team Joe Vincent (Stanford), Mac Schwager (Stanford)

Description The Reachable Polyhedral Marching (RPM) tool [47] is a method for computing exact forward and backward reachable sets of feedforward neural networks with ReLU activation. Verification problems are posed as backward reachability problems. A unique feature of the RPM tool is its incremental computation of reachable sets. For verification this means that unsafe inputs may be found before computing the complete reachable set, leading to early termination. RPM does not currently have a parallel implementation, although the algorithm is amenable to parallelization.

Link https://github.com/StanfordMSL/Neural-Network-Reach/tree/vnn_comp_2021

Commit `861ce6e380e3cc2d439a7bca87b59817e4624af6` for reproducing competition results. For other purposes the most recent commit is suggested.

Hardware and licences CPU, no license is required.

Participated benchmarks ACASXu

3.9 ComposableNeuralVerification (NV.jl)

Team Tianhao Wei (Carnegie Mellon), Chen Tan (Northeastern), Changliu Liu (Carnegie Mellon)

Description This tool is adapted from the original NeuralVerification.jl [25] developed at the Stanford Intelligent System Lab. This Julia toolbox implemented a wide variety of verification algorithms that use reachability, optimization, and search, which are summarized in [26]. We added support for onnx format networks and vnnlib format specifications,

Link <https://github.com/intelligent-control-lab/NeuralVerification.jl>

Hardware and licences CPU, no licence

Commit `4e612602ba4b34b42416742d85476d9b0dcdeb51` (for reproducing competition results; otherwise please use the master branch)

Participated benchmarks nn4sys and AcasXu

3.10 Venus

Team Panagiotis Kouvaros (Imperial College London), Alessio Lomuscio (Imperial College London)

Description Venus is a complete verification tool for Relu-based feed-forward neural networks. Venus implements a MILP-based verification method whereby it leverages dependency relations between the ReLU nodes to reduce the search space that needs to be considered during branch-and-bound. The dependency relations are exploited via callback cuts [6] and via a branching method that divides the verification problem into a set of sub-problems whose MILP formulations require fewer integrality constraints [23]. To derive tight MILP encodings, Venus additionally implements a symbolic interval propagation method for computing the pre-activation bounds of the ReLU nodes; the method optimises the linear relaxation of each of the ReLU nodes towards minimising the over-approximation error in subsequent layers.

Link <https://github.com/vas-group-imperial/venus2>

Commit For the reproduction of the VNN-COMP2021 results please use the repository https://github.com/pkouvaros/venus2_vnncomp21 (57e9608041d230b5d78c4f2afb890b81035436a1).

Hardware and licenses CPU, GUROBI License.

Participated benchmarks ACASXU, mnistfc, nn4sys.

3.11 Debona

Team Christopher Brix (RWTH Aachen University), Thomas Noll (RWTH Aachen University)

Description Debona is a fork of VeriNet [17]. However, the abstract domain used by VeriNet defines symbolic linear upper and lower bounds that are parallel to each other, i.e., offset only by some scalar value. On the contrary, Debona utilizes independent upper and lower bounds. This allows for a tighter relaxation especially for ReLU operations, where a lower bound of zero may be better than bounds that are negative in large regions of the input space. This idea has been described in [7] but was independently previously published in [38].

Link <https://github.com/ChristopherBrix/Debona>

Commit f000f3d483b2cc592233d0ba2a1a0327210562c8

Hardware and licences CPU, Gurobi licence

Participated benchmarks AcasXu, eran, mnistfc and nn4sys

3.12 nnum

Team Stanley Bak (Stony Brook Univeristy)

Description The nnum tool uses multiple levels of abstraction to achieve high-performance verification of ReLU networks without sacrificing completeness [3]. Analysis combines three types of zonotopes with star set (triangle) overapproximations [45], and uses efficient parallelized ReLU case splitting [5]. The ImageStar method [43] allows sets to be quickly propagated through all layers supported by the ONNX runtime, such as convolutional layers with arbitrary parameters. The tool is written in Python 3, uses GLPK for LP solving. New this year we added support for `vnnlib` files, and optimized some of the LP timeout parameters for acasxu [4].

Link <https://github.com/stanleybak/num>

Commit c93a39cb568f58a26015bd151acafab34d2d4929

Benchmark	Application	Network Types	Largest NN
Acasxu	Control	Feedforward + ReLU Only	54.6k
Cifar10_resnet	Image Classification	ResNet	487k
Cifar2020 (unscored)	Image Classification	Conv + ReLU	9.41M
Eran	Image Classification	Feedforward + non-ReLU	1.68M
Marabou-cifar10	Image Classification	Conv + ReLU	1.29M
Mnistfc	Image Classification	Feedforward + ReLU Only	2.03M
nn4sys	Database Indexing	Feedforward + ReLU Only	336.5M*
Oval21	Image Classification	Conv + ReLU	840k
Verivital	Image Classification	Conv + maxpool / avgpool	46.3k

*After zipping, the network is of 1.79M.

Table 2: Overview of all benchmarks.

Hardware and licences CPU, No licenses required

Participated benchmarks AcasXu, cifar2020, mnistfc, oval

4 Benchmarks

4.1 ACAS Xu

Networks The ACASXu benchmarks consists of ten properties defined over 45 neural networks used to issue turn advisories to aircraft to avoid collisions. The neural networks have 300 neurons arranged in 6 layers, with ReLU activation functions. There are five inputs corresponding to the aircraft states, and five network outputs, where the minimum output is used as the turn advisory the system ultimately produces.

Specifications We use the original 10 properties [21], where properties 1-4 are checked on all 45 networks as was done in later work by the original authors [22]. Properties 5-10 are checked on a single network. The total number of benchmarks is therefore 186. The original verification times ranged from seconds to days—including some benchmark instances that did not finish. This year we used a timeout of around two minutes (116 seconds) for each property, in order to fit within a total maximum runtime of six hours.

4.2 Cifar10_resnet

Proposed by the α, β -CROWN team.

Motivations Currently, many tools are hard-coded to handle feedforward networks only. To make neural network verification more useful in practical scenarios, we advocate that tools should handle more general architecture. Residual networks [16] (ResNet) is the first step towards this goal due to its relatively simple structure and practical significance. The propose of this benchmark is to provide some incentives for the community to develop more generic tools.

Networks We provided two ResNet models on CIFAR-10 image classification task with the following structures:

- ResNet-2B with 2 residual blocks: 5 convolutional layers + 2 linear layers

- **ResNet-4B** with 4 residual blocks: 9 convolutional layers + 2 linear layers

The networks are trained via adversarial training with an ℓ_∞ perturbation norm of $\epsilon = \frac{2}{255}$. For simplicity, these networks do not contain batch normalization or pooling layers, and use ReLU activation functions. The ResNet-4B model is relatively large with over 10K neurons.

We evaluated both networks using a 100-step projected gradient descent (PGD) attack with 5 random restarts, and a simple bound propagation based verification algorithm CROWN [55] (mathematically equivalent to the abstract interpretation used in DeepPoly [38]) under ℓ_∞ norm perturbations. The results are listed in Table 3.

Model	# ReLUs	Clean acc.	$\epsilon = 2/255$		$\epsilon = 1/255$	
			PGD acc.	Verified acc.	PGD acc.	Verified acc.
ResNet-2B	6244	69.25%	54.82%	26.88%	62.24%	57.16%
ResNet-4B	14436	77.20%	61.41%	0.24%	69.75%	23.28%

Table 3: Clean accuracy, PGD accuracy and CROWN verified accuracy for ResNet models. Note that the verified accuracy is obtained via the vanilla version of CROWN/DeepPoly which has been widely used as a simple baseline, not the α, β -CROWN tool used in the competition.

To ensure the appropriate level of difficulty, we use $\epsilon = \frac{2}{255}$ for the ResNet-2B model and $\epsilon = \frac{1}{255}$ for the ResNet-4B model.

Specifications We randomly select 48 images from the CIFAR-10 test set for the ResNet-2B model and 24 images for the ResNet-4B model. The images are classified correctly and cannot be attacked by a 100-step PGD attack with 5 random restarts. For each image, we specify the property that the logit of the ground-truth label is always greater than the logits of all other 9 labels within ℓ_∞ norm input perturbation of $\epsilon = \frac{2}{255}$ for ResNet-2B and $\epsilon = \frac{1}{255}$ for ResNet-4B. The per-example timeout is set to 5 minutes and the overall runtime is guaranteed to be less than 6 hours.

4.3 Cifar2020 (unscored)

Motivation This benchmark combines two convolutional CIFAR10 networks from last year’s VNN-COMP 2020 with a new, larger network with the goal to evaluate the progress made by the whole field of Neural Network verification.

Networks The two ReLU networks `cifar_10_2_255` and `cifar_10_8_255` with two convolutional and two fully-connected layers were trained for ℓ_∞ perturbations of $\epsilon = \frac{2}{255}$ and $\frac{8}{255}$, respectively, using COLT [29] and the larger `ConvBig` with four convolutional and three fully-connected networks, was trained using adversarial training [28] and $\epsilon = \frac{2}{255}$.

Specifications We draw the first 100 images from the CIFAR10 test set and for every network reject incorrectly classified ones. For the remaining images, the specifications describe a correct classification under an ℓ_∞ -norm perturbation of at most $\frac{2}{255}$ and $\frac{8}{255}$ for `cifar_10_2_255` and `ConvBig` and `cifar_10_8_255`, respectively and allow a per sample timeout of 5 minutes.

4.4 eran

Proposed by: The ERAN team

Motivation While most Neural Network Verification methods focus their analysis on ReLU based networks, many modern network architectures, e.g., EfficientNet [41], are based on non-piecewise-linear activation functions. To begin to understand how the choice of activation function affects certifiability, the eran benchmark aims at comparing the certifiability of networks based on piecewise-linear and non-piecewise-linear activation functions under an ℓ_∞ -norm based adversary.

Networks We consider a ReLU network with 8 hidden layers of width 200 and a Sigmoid network with 6 hidden layers of width 200. Both networks were trained using standard training.

Specifications We sample random images from the MNIST test set until we obtain 36 correctly classified images per network. For these images, the specifications describe a correct classification under an ℓ_∞ -norm perturbation of at most 0.015 and 0.012 for the ReLU and Sigmoid network, respectively, and allow a per sample timeout of 5 minutes.

4.5 Marabou-cifar10

Proposed by The Marabou team.

Networks This benchmark contains three convolutional networks, `cifar10_small.onnx`, `cifar10_medium.onnx`, and `cifar10_large.onnx`, trained on the CIFAR10 dataset. Each network has 2 convolutional layers followed by two fully connected feed-forward layers. Each layer uses the ReLU activation functions. The networks are all trained with Adam optimizer for 120 epochs with learning rate 0.0002. The three networks contain 2568, 4944, and 10528 ReLUs, respectively. The test accuracy are 63.14%, 70.21%, and 74.16%, respectively. The networks expect the input image to be normalized between 0 and 1.

Specifications We randomly sample correctly classified images from the CIFAR10 test set. The specifications are targeted adversarial robustness, which states that the network does not mis-classify an image as a given adversarial label under l_∞ -norm perturbations. The target label is chosen as $(correctLabel + 1) \bmod 10$. We propose two perturbation bounds: 0.012 and 0.024, and allow a per-query timeout of 5 minutes.

4.6 Mnistfc

Proposed by The VeriNet team.

Motivation This benchmark contains fully connected networks with ReLU activation functions and varying depths.

Networks The benchmark set consists of three fully-connected classification networks with 2, 4 and 6 layers and 256 ReLU nodes in each layer trained on the MNIST dataset. The networks were first presented in a benchmark in VNN-COMP 2020 [48].

Specifications We randomly sampled 15 correctly classified images from the MNIST test set. For each network and image, the specification was a correct classification under l_∞ perturbations of at most $\epsilon = 0.03$ and $\epsilon = 0.05$. The timeouts were 2 minutes per instance for the 2-layer network and 5 minutes for the remaining two networks.

4.7 NN4Sys

Proposed by The ComposableNeuralverification team

Application The benchmark contains networks for database indexing, which is a 1D to 1D mapping.

- *Background:* learned index is a neural network (NN) based database index proposed by Kraska et al. [24], 2018. It shows great potential but has one drawback—for non-existing keys (i.e., the keys that do not exist in the database), the outputs of a learned index can be arbitrary.
- *What we do:* to provide safety guarantees for *all* keys, we design a specification to dictate how “far” one predicted position can be, compared to its actual position (or the positions that non-existing keys should be).
- *What to verify:* our benchmark provides multiple pairs of (1) learned indexes (trained NNs) and (2) corresponding specifications. We design these pairs with different parameters such that they cover a variety of user needs and have varied difficulties for verifiers.
- *Translating learned indexes to a VNN benchmark:* the original learned index [24] contains two stages (of NNs) for high precision. However, this cascading structure is inconvenient/unsupported to verify because there is a “switch” operation—choosing one NN in the second stage based on the prediction of the first stage’s NN. To convert learned indexes to a standard form, we merge the NNs in both stages into an integrated network by adding some hand crafted layers.
- *A note on broader impact:* using NNs for systems is a broad topic, but many existing works lack strict safety guarantees. We believe that NN Verification can help system developers gain confidence to apply NNs to critical systems. We hope our benchmark can be an early step towards this vision.

Networks The networks are feedforward with ReLU activations, and they are sparse networks. There are six networks in this benchmark. Three of them have original size 194.2M and zipped size 1.79M; the other three have original size 336.5M and zipped size 790k. This is because onnx does not support directly encoding of sparse matrices, hence the networks are stored as fully connected networks.

Specifications The specification aims to check if the prediction error is bounded. The specification is a collection of pairs of input and output intervals such that any input in the input interval should be mapped to the corresponding output interval.

4.8 Oval21

Proposed by The OVAL team.

Motivations The majority of adversarial robustness benchmarks consider image-independent perturbation radii, possibly resulting in some properties that are either easily verified by all verification methods, or too hard to be verified (for commonly employed timeouts) by any of them. In line with the OVAL verification dataset from VNN-COMP 2020 [48], whose versions have already been used in various recent works [8, 11–13, 19, 20, 27, 49, 54], the OVAL21 benchmark associates to each image-network pair a perturbation radius found via binary search to ensure that all properties are challenging to solve.

Networks The benchmark includes 3 ReLU-based convolutional networks which were robustly trained [50] against ℓ_∞ perturbations of radius $\epsilon = 2/255$ on CIFAR10. Two of the networks, named **base** and **wide**, are composed of 2 convolutional layers followed by 2 fully connected layers and have respectively 3172 and 6244 activations. The third model, named **deep**, has 2 additional convolutional layers and a total of 6756 activations.

Specifications The verification properties represent untargeted adversarial robustness (with respect to all possible misclassifications) to ℓ_∞ perturbations of varying ϵ , with a per-instance timeout of 720 seconds. The property generation procedure relies on commonly employed lower and upper bounds to the adversarial loss to exclude perturbation radii that yield trivial properties. 10 correctly classified images per network are randomly sampled from the entire CIFAR10 test set, and a distinct $\epsilon \in [0, 16/255]$ is associated to each. First, a binary search is run to find the largest ϵ value for which a popular iterative adversarial attack [14] fails to find an adversarial example. Then, a second binary search is run to find the smallest ϵ value for which bounds [54] from the element-wise convex hull of the activations (with fixed intermediate bounds from [50, 55]) fail to prove robustness. Both binary search procedures are run with a tolerance of $\epsilon_{\text{tol}} = 0.1$. Denoting ϵ_{lb} as the smallest output from the two routines, and ϵ_{ub} as the largest, the following perturbation radius is chosen: $\epsilon = \frac{1}{3}\epsilon_{lb} + \frac{2}{3}\epsilon_{ub}$.

Link <https://github.com/stanleybak/vnncomp2021/tree/main/benchmarks/oval21>

4.9 Verivital

Proposed by The VeriVITAL team.

Motivation Neural networks with pooling layers are vastly used in several applications. The main motivation for proposing this benchmark was to include the pooling layers as part of this year’s VNN-Comp.

Networks This benchmark contains two MNIST classifiers with pooling layers, one with averagepooling layers and the other with maxpooling.

Specifications We randomly sampled 20 correctly classified images from the MNIST test set. For the network with averagepooling layers, the specification was to correctly classify those randomly chosen images with an ℓ_∞ perturbation radii(ϵ) of 0.02 and 0.04 and a timeout of 5 minutes. For the network with maxpooling layers the corresponding radius was 0.004 with a timeout of 7 minutes.

Link <https://github.com/stanleybak/vnncomp2021/tree/main/benchmarks/verivital>

5 Results

Each tool was run on all benchmarks which produced a `csv` file of results. This was sent to the authors for review, which sometimes required rerunning certain benchmarks to make sure they match the expected results. Python code was then written to process the results and compute the scores. The final `csv` files for each tool as well as scoring scripts are available online: https://github.com/stanleybak/vnncomp2021_results.

This also includes detailed log files for each benchmark showing the specific runtime for each tool and the score awarded. This can be used to find challenging instances to help with tool development. For example, in the output files in the repo you may see things like:

```
Row: ['ACASXU_run2a_4_2_batch_2000-prop_2', '-', '6.4 (h)', '10.5 (h)',
      'timeout', '41.1 (h)', 'timeout', 'timeout', '64.8 (h)', '62.5 (h)',
      'timeout', 'timeout', 'timeout', '-']
73: nnv score: 0
73: nnenum score: 12
73: venus2 score: 11
73: NN-R score: 0
73: VeriNet score: 10
73: DNNF score: 0
73: Debona score: 0
73: a-b-CROWN score: 10
73: oval score: 10
73: Marabou score: 0
73: ERAN score: 0
73: NV.jl score: 0
73: randgen score: 0
```

The tools are listed in order, and row is the times and result for each tool. So `nnenum` should be holds with a time of 6.4 (after subtracting overhead). The scores are also listed for each tool. Since `nnenum` was the fastest on this instance, it got 12 points, 10 for correct plus 2 for time bonus as fastest. The second fastest was `venus2` at 10.5 seconds, so they get 11 points. None of the remaining tools were within 0.2 seconds, so they all received 10 points if they completed analysis successfully.

5.1 Overall Score

The overall score for VNN-COMP 2021 is shown in Table 4. Two tables are included, based on the two ways to measure overhead discussed in Section 2. Overall, α, β -CROWN performed best, followed by VeriNet. We awarded two third place results, one to oval and one to ERAN, as their ranking depended upon factors like overhead as well as how incorrect results were judged, discussed next. For all the remaining tables in this section, the numbers reported correspond to the multi-overhead measurements.

One unexpected aspect was how to judge incorrect results, since tools currently are not required to produce a counter-example when an instance is falsified. We considered two reasonable options, which was voting (majority is assumed to be correct), and odd-one-out. In odd-one-out, only if a single tool's output differs from all the others is the result is considered incorrect. If multiple tools produce the same result or if only two tools completed the instance and their results differ, then the instance is ignored for scoring purposes. This generally had a

slight effect on the score, which was significant enough to affect the order in the total ranking. Specifically, the positions of ERAN and oval for third place could be affected by the scoring parameters, when using the "Single Overhead" overhead correction. Notice that, however, both voting and odd-one-out are imperfect ways to judge incorrect results, and it may be the case that the mismatching tool was in fact correct. In future editions of VNN-COMP we may standardize counter-example outputs and require they are produced when instances can be falsified, to remedy this shortcoming. For the reported category scores, we display odd-one-out scores and results.

Other statistics and the individual benchmark scores are also included below. In general, the GPU tools did better, as well as tools that could support a large number of benchmarks and verified a large number of benchmark instances. Several tools produced mismatching results, and an interesting followup study could identify the underlying reasons for this unsoundness.

Table 4: VNN-COMP 2021 Overall Score

(a) Single Overhead			(b) Multi-Overhead		
#	Tool	Score	#	Tool	Score
1	α, β -CROWN	779.2	1	α, β -CROWN	779.7
2	VeriNet	701.2	2	VeriNet	705.0
3	oval	582.0	3	ERAN	643.4
4	ERAN	581.1	4	oval	581.8
5	Marabou	335.3	5	Marabou	339.0
6	Debona	201.9	6	Debona	201.9
7	venus2	189.2	7	venus2	189.2
8	nenum	184.5	8	nenum	184.6
9	nnv	57.2	9	nnv	57.2
10	NV.jl	48.1	10	NV.jl	48.1
11	RPM	25.4	11	RPM	25.4
12	DNNF	24.3	12	DNNF	24.3
13	randgen	1.9	13	randgen	1.9

5.2 Other Stats

This section presents other statistics related to the measurements that are interesting, but did not play a direct role in scoring this year. With ERAN, which had different overheads, the ‘CPU Mode’ column in Table 5 corresponds to the overhead used for the ACASXu and ERAN benchmarks, whereas the ‘Seconds’ column corresponds to when the GPU was used (all others).

Table 5: Overhead

#	Tool	Seconds	CPU Mode
1	Marabou	0.2	-
2	randgen	0.3	-
3	nnenum	1.0	-
4	venus2	1.7	-
5	DNNF	2.0	-
6	VeriNet	2.2	-
7	Debona	2.5	-
8	oval	5.1	-
9	α, β -CROWN	6.1	-
10	ERAN	7.1	3.7
11	nnv	8.4	-
12	NV.jl	20.9	-
13	RPM	52.2	-

Table 6: Num Benchmarks Participated

#	Tool	Count
1	VeriNet	9
2	ERAN	9
3	α, β -CROWN	9
4	oval	8
5	Marabou	7
6	nnv	6
7	DNNF	5
8	randgen	4
9	nnenum	4
10	Debona	4
11	venus2	3
12	NV.jl	2
13	RPM	1

Table 7: Num Instances Verified

#	Tool	Count
1	α, β -CROWN	766
2	VeriNet	717
3	ERAN	656
4	oval	636
5	Marabou	364
6	mnenum	310
7	Debona	280
8	venus2	266
9	nnv	141
10	NV.jl	97
11	RPM	72
12	DNNF	55
13	randgen	33

Table 8: Num Violated

#	Tool	Count
1	ERAN	177
2	α, β -CROWN	177
3	oval	175
4	VeriNet	175
5	Marabou	103
6	mnenum	73
7	Debona	70
8	venus2	63
9	DNNF	55
10	RPM	44
11	randgen	33
12	NV.jl	11

Table 9: Num Holds

#	Tool	Count
1	α, β -CROWN	589
2	VeriNet	542
3	ERAN	479
4	oval	461
5	Marabou	261
6	nenum	237
7	Debona	210
8	venus2	203
9	nnv	141
10	NV.jl	86
11	RPM	28

Table 10: Mismatched (Incorrect) Results

#	Tool	Count
1	Marabou	26
2	Debona	14
3	nnv	12
4	NV.jl	5
5	venus2	1

5.3 Benchmark Scores

The results for the individual categories are shown below. For overall score, the tools which participated in all or almost all of the benchmarks did best. Within individual benchmarks, some tools performed well despite not ranking high in the overall score. The results presented here are for multi-overhead setup, with incorrect results scored using the odd-one-out strategy. Adjusting these parameters produced minor changes in the overall score and rankings, but was omitted for clarity. If these alternate scores are of interest, the discussion at the beginning of Section 5 outlines where to access the scripts used to compute scores.

Table 11: Benchmark `acasxu`

#	Tool	Verified	Falsified	Fastest	Score	Percent
1	nnum	138	47	155	1910	100.0%
2	VeriNet	138	47	117	1852	97.0%
3	Marabou	137	46	115	1809	94.7%
4	oval	138	47	98	1794	93.9%
5	venus2	138	46	94	1778	93.1%
6	α, β -CROWN	138	47	67	1732	90.7%
7	ERAN	125	46	24	1506	78.8%
8	Debona	84	42	39	1086	56.9%
9	RPM	28	44	9	486	25.4%
10	nnv	29	0	29	348	18.2%
11	DNNF	0	41	12	182	9.5%
12	randgen	0	28	0	28	1.5%
13	NV.jl	45	9	0	-23	0%

Table 12: Benchmark `cifar10-resnet`

#	Tool	Verified	Falsified	Fastest	Score	Percent
1	α, β -CROWN	58	0	12	623	100.0%
2	VeriNet	48	0	29	548	88.0%
3	ERAN	43	0	36	502	80.6%
4	Marabou	39	0	0	390	62.6%

Table 13: Benchmark `cifar2020`

#	Tool	Verified	Falsified	Fastest	Score	Percent
1	oval	146	41	174	2209	100.0%
2	α, β -CROWN	148	42	43	1996	90.4%
3	VeriNet	139	42	5	1822	82.5%
4	ERAN	107	43	132	1749	79.2%
5	nnenum	62	13	0	741	33.5%
6	randgen	0	2	0	2	0.1%
7	nnv	6	0	0	-140	0%

Table 14: Benchmark `eran`

#	Tool	Verified	Falsified	Fastest	Score	Percent
1	α, β -CROWN	60	1	26	670	100.0%
2	VeriNet	48	1	49	588	87.8%
3	ERAN	46	1	0	470	70.1%
4	Debona	47	2	39	375	56.0%
5	oval	21	0	18	247	36.9%
6	Marabou	19	0	0	190	28.4%
7	nnv	10	0	0	100	14.9%

Table 15: Benchmark `marabou-cifar10`

#	Tool	Verified	Falsified	Fastest	Score	Percent
1	α, β -CROWN	1	52	52	625	100.0%
2	ERAN	0	52	51	613	98.1%
3	oval	0	53	44	611	97.8%
4	VeriNet	0	52	16	543	86.9%
5	Marabou	0	29	0	281	45.0%
6	DNNF	0	2	0	11	1.8%
7	randgen	0	1	0	1	0.2%

Table 16: Benchmark `mnistfc`

#	Tool	Verified	Falsified	Fastest	Score	Percent
1	α, β -CROWN	49	21	35	772	100.0%
2	VeriNet	39	21	57	716	92.7%
3	Debona	37	22	47	688	89.1%
4	oval	37	21	46	676	87.6%
5	ERAN	34	22	47	654	84.7%
6	Marabou	35	19	0	540	69.9%
7	venus2	31	16	26	522	67.6%
8	nnenum	35	12	21	512	66.3%
9	nnv	27	0	8	186	24.1%
10	DNNF	0	7	0	70	9.1%

Table 17: Benchmark `mn4sys`

#	Tool	Verified	Falsified	Fastest	Score	Percent
1	α, β -CROWN	70	5	73	878	100.0%
2	VeriNet	68	3	0	719	81.9%
3	ERAN	67	4	0	704	80.2%
4	oval	56	4	0	589	67.1%
5	NV.jl	41	2	0	422	48.1%
6	venus2	34	1	0	250	28.5%
7	DNNF	0	4	0	22	2.5%
8	randgen	0	2	0	2	0.2%
9	Debona	42	4	0	-722	0%

Table 18: Benchmark `ova121`

#	Tool	Verified	Falsified	Fastest	Score	Percent
1	oval	12	2	11	164	100.0%
2	α, β -CROWN	12	2	2	146	89.0%
3	VeriNet	11	2	6	145	88.4%
4	ERAN	6	2	3	86	52.4%
5	Marabou	4	2	1	63	38.4%
6	nnenum	2	1	0	30	18.3%
7	nnv	16	0	4	-31	0%

Table 19: Benchmark verivital

#	Tool	Verified	Falsified	Fastest	Score	Percent
1	α, β -CROWN	53	7	52	704	100.0%
2	oval	51	7	57	694	98.6%
3	ERAN	51	7	56	693	98.4%
4	VeriNet	51	7	0	580	82.4%
5	DNNF	0	1	0	10	1.4%
6	nnv	53	0	0	-168	0%
7	Marabou	27	7	0	-2260	0%

6 Conclusion and Ideas for Future Competitions

This report summarizes the 2nd Verification of Neural Networks Competition (VNN-COMP) held in 2021. Improvements to the competition structure have been made, including standardization of common input formats (`onnx` and `vnnlib`), and common measurement hardware. Based on the common benchmarks (CIFAR2020 and ACASXU), tools exhibited significant progress in terms of scalability and speed compared with previous years. The comparison is imperfect, as last year we did not have standardized hardware, so it is unclear how much of the speed improvement is due to algorithmic improvements and how much is due to better hardware. Future editions of the competition may better judge the year-to-year improvements in neural network verification methods.

The benchmarks and tool execution scripts are openly available for others to replicate: <https://github.com/stanleybak/vnncomp2021>. We hope this serves as a fair comparison for evaluating future improvements to verification methods in upcoming publications. From an applicability perspective, having a common input format hopefully reduces the barriers of industry participants to use the developed tools.

In future editions of VNN-COMP, some improvements we can make would be to allow more types of hardware. For example, any AWS EC2 machine could be chosen by the tool authors with roughly the same cost (\$3 an hour this year). Multiple tool authors expressed that their tool could work faster if given both a strong CPU and GPU together. Alternatively, we could allow custom hardware per benchmark. This would likely require additional automation with the competition measurements, especially dealing with installing license files. Some ideas for automating this could be to have authors provide the Gurobi licences to use, rather than the competition organizers. Another improvement would be to improve overhead measurement, as detailed in Section 2. Finally, we should standardize the counter-example format, so that the ground truth for mismatched verification results can be provided.

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Tool authors listed in section 3 participated in the preparation and review of this report.

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